Annuities Galore on the HP-12C

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Example 1: 10 [n] 5 [i] [R/S] → 7.722. [RCL] 1 → 39.374 ... [RCL] 5 → 169.396, six annuities in all. For j=0 to 3, R_j values the sequence t^j at time t, where t=1 to n. R_4 values (n-t+1) and R_5 t*(n-t+1). See below for more detail.

Keystrokes	Display	Keystrokes	Display	Keystrokes	Display
f P/R		+	32- 40	RCL n	65-45 11
f CLEAR PRGM	00-	%T	33- 23	STO 0	66-44 0
RCL i	01-45 12	STO 5	34-44 5	ENTER	67- 36
g x=0	02-43 35	R↓	35- 33	×	68- 20
g GTO 65	03-43,33 65	RCL 2	36-45 2	g LSTx	69-43 36
RCL n	04-45 11	x≷y	37- 34	+	70- 40
1	05- 1	%	38- 25	2	71- 2
CHS	06- 16	+	39- 40	÷	72- 10
PMT	07- 14	RCL n	40-45 11	STO 1	73-44 1
CLx	08- 35	ENTER	41- 36	STO 3	74-44 3
g END	09-43 8	×	42- 20	STO × 3	75-44 20 3
FV	10- 15	CHS	43- 16	STO 4	76-44 4
PV	11- 13	FV	44- 15	RCL n	77-45 11
STO 0	12-44 0	PV	45- 13	×	78- 20
−	13- 30	−	46- 30	RCL 1	79-45 1
%T	14- 23	%T	47- 23	+	80- 40
STO 4	15-44 4	STO 2	48-44 2	RCL n	81-45 11
R↓	16- 33	RCL 1	49-45 1	ENTER	82- 36
RCL n	17-45 11	−	50- 30	+	83- 40
g BEG	18-43 7	x≷y	51- 34	1	84- 1
FV	19- 15	%	52- 25	+	85- 40
PV	20- 13	+	53- 40	3	86- 3
%T	21- 23	3	54- 3	÷	87- 10
STO 1	22-44 1	×	55- 20	RCL 1	88-45 1
ENTER	23- 36	RCL n	56-45 11	×	89- 20
+	24- 40	g LSTx	57-43 36	STO 2	90-44 2
STO 2	25-44 2	y^x	58- 21	−	91- 30
CHS	26- 16	FV	59- 15	STO 5	92-44 5
RCL PMT	27-45 14	PV	60- 13	RCL 0	93-45 0
FV	28- 15	+	61- 40	g GTO 00	94-43,33 00
PV	29- 13	%T	62- 23	f P/R	
RCL n	30-45 11	STO 3	63-44 3		
×	31- 20	g GTO 93	64-43,33 93		

The formulae are:

Annuity Type	**i<>0, v=1/(1+i)**	**i=0**
Level, of 1 in arrears	$R_0=(1-v^n)/i$	$R_0=n$
Increasing, of 1,2, ..., n	$R_1=((1+i)R_0 - nv^n)/i$	$R_1=n(n+1)/2$
Increasing, of 1,4, ..., n^2	$R_2=(2(1+i)R_1-(1+i)R_0-n^2v^n)/i$	$R_2=R_1(2n+1)/3$
Increasing, of 1,8, ..., n^3	$R_3=(3(1+i)(R_2-R_1)+(1+i)R_0-n^3v^n)/i$	$R_3={R_1}^2$
Decreasing, of n,n-1,...,1	$R_4=(n-R_0)/i$	$R_4=R_1$
Incr./Decr., of n,2(n-1),...,n	$R_5=(n(1+i)R_0+ nv^n - 2R_1)/i$	$R_5=R_1(n+1)-R_2$

The 12cp needs 8 extra lines. Each of the five [FV][PV] sequences *needs* to be [FV][R↓][PV][PV]. *Optionally*, [x^2] can be used to replace the two [ENTER][X] (lines 41/42 and 67/68). This changes line numbers so the [GTO] need attending to.

i% is kept in the upper stack so that the "/i" is simply accomplished using %T. For i<>0 the order of calculation is R_0, R_4, R_1, R_5, R_2 and R_3 at lines 12, 15, 22, 34, 48 and 63 respectively. For i=0 the order is: R_0, R_1, R_3, R_4, R_2 and R_5. Next we tabulate the results for: 10[n] 0[i] [R/S] & 10[i][R/S] and calculate durations and demonstrate the little known but provable fact that the true equated time is very close to the arithmetic average of the approximate equated time (D_0) and the duration (D_1).

n=10	R_0	R_1	R_2	R_3	R_4	R_5
i%=0, V_0	10	55	285	3025	55	220
$D_0=R_{i+1}/R_i$	5.5	7	7.857	n/a	4	n/a
i%=10, V_1	6.145	29.036	185.656	1380.636	38.554	133.739
$D_1=R_{i+1}/R_i$	4.725	6.394	7.437	n/a	3.469	n/a
t_e *	5.110	6.702	7.652	8.230	3.727	5.222
$(D_0+D_1)/2$	5.113	6.697	7.647	n/a	3.735	n/a

*Equated time=t_e= $LN(V_0/V_1)/LN(1+i)$. $V_1=V_0(1+i)^{-te}$. $V_1\approx V_0(1+i)^{-D0}$

Suppose a firm pays every employee reaching 10 years service £1,000, and every year 10 employees do so. A £100,000 fund earning 10% would fund this forever. However the accountant suggests that only a 10 year liability needs be held, as otherwise the firm is reserving for future employees. Using the R_0=6.145 from above, this would be £61,450. On second thoughts, using accrual accounting only the next year's £10,000 need be reserved in full, as it is fully accrued. We need only reserve for 9/10th of the following £10,000, etc. This is a decreasing annuity, and we can use R_4=38.554, giving an accrued liability of only £38,554. If the firm decides to inflate the £1,000 by say 3% a year then all we need to do is use an interest rate of about 7%, or more precisely: 10[ENTER]3[−]1[g][LSTx][%][+][÷]→ 6.796[i][R/S]→7.090. [RCL]4 → 42.815. Hence the accrued liability would increase to £42,815, an increase of 11.052%. The duration from above is D=3.469, so we'd expect a 10%-6.796%=3.204% reduction in yield to increase the value by 3.469*3.204%=11.115%, surprisingly close for a 3.2% (relatively large) change!